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Effect of Background Evolution on the Curvaton Non-Gaussianity

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Abstract

We investigate how the background evolution affects the curvature perturbations generated by the curvaton, assuming a curvaton potential that may deviate slightly from the quadratic one, and parameterizing the background fluid density as $\rho \propto a^{-\alpha}$, where a is the scale factor, and α depends on the background fluid. It turns out that the more there is deviation from the quadratic case, the more pronounced is the dependence of the curvature perturbation on α. We also show that the background can have a significant effect on the nonlinearity parameters $f_{\rm NL}$ and $g_{\rm NL}$. As an example, if at the onset of the curvaton oscillation there is a dimension 6 contribution to the potential at 5 % level and the energy fraction of the curvaton to the total one at the time of its decay is at 1%, we find variations $\Delta f_{\rm NL} \sim \mathcal{O}(10)$ and $\Delta g_{\rm NL} \sim \mathcal{O}(10^4)$ between matter and radiation dominated backgrounds. Moreover, we demonstrate that there is a relation between $f_{\rm NL}$ and $g_{\rm NL}$ that can be used to probe the form of the curvaton potential and the equation of state of the background fluid.

1 Introduction

Although quantum fluctuations of the inflaton are often taken to be responsible for the origin of density perturbations, other mechanisms such as the curvaton [1–3], where fluctuations of a scalar field other than the inflaton generate primordial perturbations, have also attracted much attention recently. In particular, in the light of the recent result from WMAP5 which suggests that primordial non-Gaussianity may be large [4, 5][#1], the curvaton mechanism may be attractive since a large primordial non-Gaussianity can be generated in this scenario [6–20][#2], whereas simplest inflation models predict a non-linearity parameter $f_{\rm NL}$ that is of the order of the slow-roll parameters (or O(1) at most), and hence practically imply a Gaussian perturbation.

Usually the curvaton potential is assumed to be quadratic. However, there is no other reason for this except simplicity, and in fact in any realistic particle physics model the curvaton can be expected to have some self-interactions. Thus, deviations from the exact quadratic form are also worth investigating. They have been discussed in [8, 15–17, 19–22], where it has been pointed out that non-quadratic contributions to the potential can modify the resultant curvaton perturbations in a significant manner. In particular, the prediction for the non-linearity parameters $f_{\rm NL}$ and $g_{\rm NL}$ can change considerably as compared to the quadratic case.

In addition, there is yet another assumption which is tacitly adopted in the curvaton literature: the background evolution of the universe is determined by radiation. With this assumption, the curvaton starts to oscillate during a radiation-dominated (RD) epoch. If the curvaton decays before dominating the Universe, radiation is always the dominant component and controls the background evolution of the Universe. However, it is also possible that after inflation, the inflaton is oscillating around the minimum of the potential for a while and that the curvaton begins to oscillate during such epoch. In this case, the background evolution is different from the case of radiation and is determined by a matter-like component if the inflaton potential is approximatively quadratic[#3]. After inflation there could also exist a possibility of a kination-dominated phase where the kinetic term of some scalar field can dominate the energy density of the universe. Such fluid has a stiff equation of state with $w = 1$ while its energy density decreases as $\rho \propto a^{-6}$.

In this paper, we investigate how the background evolution of the universe affects the curvature perturbation generated from the curvaton. We first show that when the curvaton potential has a quadratic form, the background evolution has little effect on the curvature perturbation. However, when the curvaton potential includes a non-quadratic term, the nonlinear evolution of the curvaton field is affected much by the background, and the

[#1] The degree of non-Gaussianity is usually characterized by the lowest order non-linearity parameter $f_{\rm NL}$. The current constraint on $f_{\rm NL}$ is $-9 < f_{\rm NL} < 111$ in Ref. [4] or $-4 < f_{\rm NL} < 80$ at 95% C.L. in Ref. [5]. Although purely Gaussian fluctuations with $f_{\rm NL} = 0$ are allowed, the central value is away from zero.

[#2] Other models, for example, such as the modulated reheating scenario [23, 24] are also known to generate large non-Gaussian fluctuations [25–28].

[#3] In Ref. [18], this kind of situation is also included in the analysis for the quadratic curvaton potential.

resultant curvature perturbations can be significantly modified from the usual RD case. We investigate this issue by assuming a general background fluid and a potential that slightly deviates from a quadratic form, and derive the dependence of the curvature perturbation and/or the non-linearity parameters such as $f_{\rm NL}$ and $g_{\rm NL}$ on different background fluids.

The structure of the paper is as follows. In the next section, we summarize the formalism and give definitions of the quantities required for the subsequent analysis. In section 3, we discuss how the background evolution affects the quantities such as power spectrum and non-linearity parameters, using the appropriate formulas for arbitrary background fluids. The final section is devoted to a summary and a discussion of the results.

2 Formalism and Definitions

Let us begin by summarizing the formalism and the definitions of the various quantities required for the subsequent discussion. Here we consider a potential of the curvaton field σ which, in addition to the usual quadratic term, also includes a non-renormalizable term:

$$V(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \lambda m_\sigma^4\left(\frac{\sigma}{m_\sigma}\right)^n , \tag{1}$$

where m_σ is the mass of the curvaton, and λ is a constant. For the purpose of this paper, it is enough to investigate the case of a slight deviation from the purely quadratic form. Thus in the following we assume that the quadratic term always dominates over the non-quadratic term (for a general discussion of the ramifications of non-quadratic terms in curvaton models, see [21, 22]). We characterize the relative contribution of the non-quadratic term at the time when the curvaton is still in a slowly-rolling regime by the parameter s, defined as

$$s \equiv 2\lambda\left(\frac{\sigma_*}{m_\sigma}\right)^{n-2} . \tag{2}$$

To investigate the curvature perturbation ζ generated by the curvaton field, we adopt the δN formalism [29–32] and calculate ζ up to the third order as

$$\zeta = \frac{dN}{d\sigma_*}\delta\sigma_* + \frac{1}{2}\frac{d^2N}{d\sigma_*^2}(\delta\sigma_*)^2 + \frac{1}{6}\frac{d^3N}{d\sigma_*^3}(\delta\sigma_*)^3 + \cdots . \tag{3}$$

Once we obtain ζ up to the third order, the power spectrum P_ζ, bispectum B_ζ, and trispectrum T_ζ are given by

$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle = (2\pi)^3 P_\zeta(k_1)\delta(\vec{k}_1+\vec{k}_2), \tag{4}$$

$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = (2\pi)^3 B_\zeta(k_1,k_2,k_3)\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3). \tag{5}$$

$$\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4}\rangle = (2\pi)^3 T_\zeta(k_1,k_2,k_3,k_4)\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3+\vec{k}_4), \tag{6}$$

where B_ζ and T_ζ can be written as

$$B_\zeta(k_1,k_2,k_3) = \frac{6}{5} f_{\rm NL} \left(P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1) \right), \tag{7}$$

$$\begin{aligned} T_\zeta(k_1,k_2,k_3,k_4) = & \ \tau_{\rm NL} \left(P_\zeta(k_{13})P_\zeta(k_3)P_\zeta(k_4) + 11 \text{ perms.} \right) \\ & + \frac{54}{25} g_{\rm NL} \left(P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4) + 3 \text{ perms.} \right). \end{aligned} \tag{8}$$

Here $f_{\rm NL}, \tau_{\rm NL}$ and $g_{\rm NL}$ are non-linearity parameters often used in the literature. Note that, in our case, $\tau_{\rm NL}$ is related to $f_{\rm NL}$ by

$$\tau_{\rm NL} = \frac{36}{25} f_{\rm NL}^2 \ . \tag{9}$$

Thus, alternatively, we may assume the following expansion as the definition of $f_{\rm NL}$ and $g_{\rm NL}$

$$\zeta = \zeta_1 + \frac{3}{5} f_{\rm NL} \zeta_1^2 + \frac{9}{25} g_{\rm NL} \zeta_1^3 + \cdots , \tag{10}$$

where ζ_1 denotes the curvature perturbation at linear order.

Let us now derive the formulae for the power spectrum, $f_{\rm NL}$ and $g_{\rm NL}$ for a general background equation of state. As we mentioned in the Introduction, usually one assumes that the curvaton oscillates in a radiation background evolution. However, it is obvious that the inflaton can be expected to oscillate around the minimum of its (quadratic) potential for some time before it decays. For a weakly coupled inflaton field, the duration of this epoch could be fairly long. During that time the inflaton behaves like matter, and it is possible that the curvaton oscillations begin during this epoch.

Eventually the inflaton will decay into radiation; this can well happen before the curvaton oscillations dominate the energy budget of the universe, or before the curvaton decay, whichever happens first. Hence the universe becomes radiation-dominated so that the background evolution for oscillating curvaton is controlled first by a matter component, followed by radiation domination. Having this kind of situation in mind, let us assume there is a transition in the background from one fluid to another one at time $t_{\rm tr}$. We then parametrize the energy density of the background fluid before the transition time by

$$\rho_{\rm BG} \propto a^{-\alpha}, \tag{11}$$

whereas after the transition, for $t > t_{\rm tr}$, we have

$$\rho_{\rm BG} \propto a^{-\beta}. \tag{12}$$

For the case of oscillating inflaton background, the transition epoch $t_{\rm tr}$ corresponds to the time when the inflaton decays. In this case, for $t < t_{\rm tr}$, $\alpha = 3$ while for $t > t_{\rm tr}$, $\beta = 4$. Another example is a universe that is first kination-dominated so that the kinetic energy of a scalar field dominates the energy density of the universe. Then, after some time,

the universe becomes radiation-dominated. In this case, $\alpha = 6$ for $t < t_{\rm tr}$ while after the transition time $\beta = 4$.

With this parametrization, we obtain the curvature perturbation at the linear order as

$$\zeta_1 = \frac{2\sigma'_{\rm ocs}}{3\sigma_{\rm osc}} R\delta\sigma \ , \tag{13}$$

where

$$R \equiv r_{\rm dec}\left(1 - kr_{\rm tr}\right) + kr_{\rm tr}. \tag{14}$$

The prime denotes the derivative with respect to σ_*. Here $r_{\rm dec}$ and $r_{\rm tr}$ roughly correspond to the fraction of energy density of the curvaton to the total energy density at the time of the curvaton decay and the transition of the background, respectively. Their precise definitions are

$$r_{\rm tr} \equiv \left.\frac{3\rho_\sigma}{\alpha\rho_{\rm BG} + 3\rho_\sigma}\right|_{\rm tr}, \qquad r_{\rm dec} \equiv \left.\frac{3\rho_\sigma}{\beta\rho_{\rm BG} + 3\rho_\sigma}\right|_{\rm dec}. \tag{15}$$

Furthermore, k is defined by

$$k = 1 - \frac{\alpha}{\beta}. \tag{16}$$

By calculating the curvature perturbation up to the third order, we find that the non-linearity parameters $f_{\rm NL}$ and $g_{\rm NL}$ read as

$$f_{\rm NL} = \frac{5}{6}\left[\frac{3}{2R}\left(\frac{\sigma''_{\rm osc}\sigma_{\rm osc}}{\sigma'^2_{\rm osc}} - 1\right) + \frac{3}{2}\frac{\sigma_{\rm osc}}{R^2}\frac{dR}{d\sigma_{\rm osc}}\right], \tag{17}$$

$$g_{\rm NL} = \frac{25}{54}\left[\frac{9}{4R^2}\left(\frac{\sigma'''_{\rm osc}\sigma^2_{\rm osc}}{(\sigma')^3_{\rm osc}} - 3\frac{\sigma''_{\rm osc}\sigma_{\rm osc}}{(\sigma'_{\rm osc})^2} + 2\right) + \frac{9\sigma_{\rm osc}}{2R^3}\frac{dR}{d\sigma_{\rm osc}}\left(\frac{3}{2}\frac{\sigma''_{\rm osc}\sigma_{\rm osc}}{(\sigma'_{\rm osc})^2} - 1\right) + \frac{9\sigma^2_{\rm osc}}{4R^3}\frac{d^2R}{d\sigma^2_{\rm osc}}\right], \tag{18}$$

Here the derivatives of R with respect to $\sigma_{\rm osc}$ appear. They are given in terms of the derivatives of $r_{\rm dec}$ and $r_{\rm tr}$ with respect to $\sigma_{\rm osc}$ as

$$\frac{dR}{d\sigma_{\rm osc}} = (1 - kr_{\rm tr})\frac{dr_{\rm dec}}{d\sigma_{\rm osc}} + k(1 - r_{\rm dec})\frac{dr_{\rm tr}}{d\sigma_{\rm osc}}, \tag{19}$$

and

$$\frac{d^2R}{d\sigma^2_{\rm osc}} = (1 - kr_{\rm tr})\frac{d^2r_{\rm dec}}{d\sigma^2_{\rm osc}} - 2k\frac{dr_{\rm dec}}{d\sigma_{\rm osc}}\frac{dr_{\rm dec}}{d\sigma_{\rm osc}} + k(1 - r_{\rm dec})\frac{d^2r_{\rm tr}}{d\sigma^2_{\rm osc}}. \tag{20}$$

The derivatives $dr_{\rm dec}/d\sigma_{\rm osc}$ and $dr_{\rm tr}/d\sigma_{\rm osc}$ are explicitly given by

$$\frac{dr_{\rm dec}}{d\sigma_{\rm osc}} = \frac{2}{3\sigma_{\rm osc}} r_{\rm dec}(1 - r_{\rm dec})[3 + (\beta - 3)r_{\rm dec}](1 - kr_{\rm tr}), \tag{21}$$

$$\frac{dr_{\rm tr}}{d\sigma_{\rm osc}} = \frac{2}{3\sigma_{\rm osc}} r_{\rm tr}(1 - r_{\rm tr})[3 + (\alpha - 3)r_{\rm tr}] \tag{22}$$

The second derivatives of $r_{\rm dec}$ and $r_{\rm tr}$ with respect to $\sigma_{\rm osc}$ can be derived by differentiating the above equations and they are calculated as

$$\frac{d^2 r_{\rm dec}}{d\sigma_{\rm osc}^2} = \frac{2}{3\sigma_{\rm osc}}\frac{dr_{\rm dec}}{d\sigma_{\rm osc}}\left[-\frac{3}{2} + (1-2r_{\rm dec})\{3+(\beta-3)r_{\rm dec}\}(1-kr_{\rm tr}) \right.$$
$$\left. + r_{\rm dec}(1-r_{\rm dec})(\beta-3)(1-kr_{\rm tr})\right] - \frac{2k}{3\sigma_{\rm osc}}\frac{dr_{\rm tr}}{d\sigma_{\rm osc}} r_{\rm dec}(1-r_{\rm dec})\{3+(\beta-3)r_{\rm dec}\}, \tag{23}$$

$$\frac{d^2 r_{\rm tr}}{d\sigma_{\rm osc}^2} = \frac{2}{3\sigma_{\rm osc}}\frac{dr_{\rm tr}}{d\sigma_{\rm osc}}\left[-\frac{3}{2} + (1-2r_{\rm tr})\{3+(\alpha-3)r_{\rm tr}\} + (\alpha-3)r_{\rm tr}(1-r_{\rm tr})\right]. \tag{24}$$

We can now write down $f_{\rm NL}$ and $g_{\rm NL}$ as functions of the parameters $\alpha, \beta, r_{\rm dec}$ and $r_{\rm tr}$ in an explicit way, although in general the expressions are very complicated. In fact, as far as we consider only the cases with $\alpha, \beta > 3$, $r_{\rm tr}$ should always be smaller than $r_{\rm dec}$ and in most cases we may assume that $r_{\rm tr} \ll r_{\rm dec}$. In this case, $R \simeq r_{\rm dec}$ and $\zeta_1, f_{\rm NL}$ and $g_{\rm NL}$ may approximately be written as

$$\zeta_1 = \frac{2}{3} r_{\rm dec}\frac{\sigma'_{\rm osc}}{\sigma_{\rm osc}}\delta\sigma_*, \tag{25}$$

$$f_{\rm NL} = \frac{5}{4r_{\rm dec}}\left(1+\frac{\sigma_{\rm osc}\sigma''_{\rm osc}}{\sigma'^2_{\rm osc}}\right) + \frac{5}{6}(\beta-6) - \frac{5r_{\rm dec}}{6}(\beta-3), \tag{26}$$

$$g_{\rm NL} = \frac{25}{54}\left[\frac{9}{4r_{\rm dec}^2}\left(\frac{\sigma_{\rm osc}^2\sigma'''_{\rm osc}}{\sigma'^3_{\rm osc}} + 3\frac{\sigma_{\rm osc}\sigma''_{\rm osc}}{\sigma'^2_{\rm osc}}\right) + \frac{9}{2r_{\rm dec}}(\beta-6)\left(1+\frac{\sigma_{\rm osc}\sigma''_{\rm osc}}{\sigma'^2_{\rm osc}}\right)\right.$$
$$\left. +\frac{1}{2}\left(189-63\beta+4\beta^2-9(\beta-3)\frac{\sigma_{\rm osc}\sigma''_{\rm osc}}{\sigma'^2_{\rm osc}}\right) - 5r_{\rm dec}(18-9\beta+\beta^2) + 3r_{\rm dec}^2(\beta-3)^2\right]. \tag{27}$$

As one can notice from these expressions, when $r_{\rm dec}$ is small, which is the interesting case because then non-Gaussianity can be large, the effects of the background are almost encoded in the changes of $\sigma_{\rm osc}$ and its derivatives. The quantities such as $\sigma_{\rm osc}, \sigma'_{\rm osc}$ and $\sigma''_{\rm osc}$ are evaluated at the beginning of the curvaton oscillation, and thus the background evolution after the transition is irrelevant if we assume a nearly quadratic potential for the curvaton. Thus in this case, after its change the background evolution does not affect much the curvature perturbation but modifies ζ_1 and its non-linearity parameters at most of order $\mathcal{O}(1)$. This can be seen directly from the above expressions: β affects the coefficients by $\mathcal{O}(1)$.

If there is no change in the background evolution up to or until after the time at which the curvaton decays, the expressions for the curvature perturbation and non-linearity parameters are quite similar to the case with $r_{\rm tr} \ll r_{\rm dec}$ mentioned above. In this case, we can simply set $k=0$ and $R = r_{\rm dec}$ in the above equations. Then $\zeta_1, f_{\rm NL}$ and $g_{\rm NL}$ can be found by replacing β with α in Eqs. (25), (26) and (27).

3 Background evolution and non-Gaussianity

Let us now apply the formalism of the previous Section and discuss how the background evolution affects the amplitude of the curvature perturbation and its non-linearity parameters. As we pointed out, the transition in the background does not affect the perturbation much if the transition occurs much before curvaton decay. Hence here we focus on the case of no transition in the background, i.e., a single fluid controls the background evolution from the time well before the curvaton begins to oscillate to well after the curvaton has decayed.

When there is no transition in the background evolution we can adopt the formulae Eqs. (25), (26) and (27) by replacing β by α in the equations, as mentioned above. To appreciate the impact of background evolution on the curvature perturbation and its nonlinearity, we plot $\zeta_1, f_{\rm NL}$ and $g_{\rm NL}$ in Figs. 1–8. In Fig. 1, the values of $f_{\rm NL}$ and $g_{\rm NL}$ are shown as a function of n (see Eq. (2) for the definition of the dimension of the non-quadratic contribution to the potential) for the cases of $\alpha = 3, 4$ and 6 which correspond to matter, radiation and kination background, respectively, in our notation. As seen from the figures, the larger the non-quadratic power n is, the more pronounced the effect of the background becomes.

To observe the magnitude of the effect more quantitatively, the values of $\zeta_1, f_{\rm NL}$ and $g_{\rm NL}$ relative to those for the case with $\alpha = 4$ and 3 are plotted in Figs. 2–4 for several values of n as a function of α. In these figures, we fix the relative strength of the non-quadratic part s and $r_{\rm dec}$ to $s = 0.05$ and $r_{\rm dec} = 0.01$. In Figs. 5–7 we also plot the same information but this time for several values of s, fixing $n = 8$ and $r_{\rm dec}$.

Interestingly, when the potential of the curvaton is close to a purely quadratic one, the background evolution does not affect the results much. However, whenever the potential deviates from the quadratic form, the background tends to suppress the curvature perturbation ζ_1 the more non-quadratic the potential is. It should also be noted that when α increases, or the background fluid becomes more stiff, ζ_1 becomes suppressed. For fixed s and n, the stiffening of the background fluid drives $f_{\rm NL}$ and $g_{\rm NL}$ to increasingly negative territory. A point worth stressing is that these modifications are not small but e.g. by changing of the radiation dominated background to matter dominated one induces shifts in the non-linearity parameters that in principle could be easily observable.

Finally, let us comment on the relation between $f_{\rm NL}$ and $g_{\rm NL}$. As pointed out in [15], when the potential of the curvaton has a purely quadratic form and $r_{\rm dec}$ is small, $f_{\rm NL}$ and $g_{\rm NL}$ are related as

$$g_{\rm NL} \simeq -\frac{10}{3} f_{\rm NL}. \tag{28}$$

However, when the potential deviates from a quadratic form, "the consistency relation" between $f_{\rm NL}$ and $g_{\rm NL}$ becomes

$$g_{\rm NL} \simeq \frac{3}{2} f_{\rm NL}^2 \left(\frac{\sigma_{\rm osc}^2 \sigma_{\rm osc}'''}{\sigma_{\rm osc}'^3} + 3 \frac{\sigma_{\rm osc} \sigma_{\rm osc}''}{\sigma_{\rm osc}'^2} \right) \left(1 + \frac{\sigma_{\rm osc} \sigma_{\rm osc}''}{\sigma_{\rm osc}'^2} \right)^{-1}, \tag{29}$$

where $r_{\rm dec} \ll 1$ is assumed. Because of the nonlinear evolution of σ, which is encoded in $\sigma_{\rm osc}$ and its derivatives appearing on the right hand side of Eq. (29), the nonlinearity parameters are now related as $-g_{\rm NL} \propto f_{\rm NL}^2$. This is in sharp contrast to the quadratic case where $-g_{\rm NL} \propto f_{\rm NL}$. Furthermore, since the coefficient of the relation depends on the nonlinear evolution of σ, it can be affected by the form of the potential and the background evolution. In this respect, the comparison of $f_{\rm NL}$ and $-g_{\rm NL}$, if they ever are observed[#4], can be very useful for probing the form of the curvaton potential and the equation of state of the background during the time when the curvaton fluctuations generate the curvature perturbation.

To demonstrate this explicitly, in Fig. 8 we plot the value of $g_{\rm NL}$ as a function of $f_{\rm NL}$ for several values of α with $n = 8$ and $s = 0.02$. For reference, we also plot the case of the pure quadratic potential with RD background ($\alpha = 4$). When $f_{\rm NL}$ is large, which corresponds to the case of $r_{\rm dec} \ll 1$, the above relations hold. As a consequence, as one can see in Fig. 8, there is a definite difference in the $g_{\rm NL} - f_{\rm NL}$ relation for different backgrounds even when the non-quadratic term in the curvaton potential remains the same. Thus the trispectrum may also provide important information not only about the curvaton potential but also about the equation of state of the background fluid during curvaton oscillations.

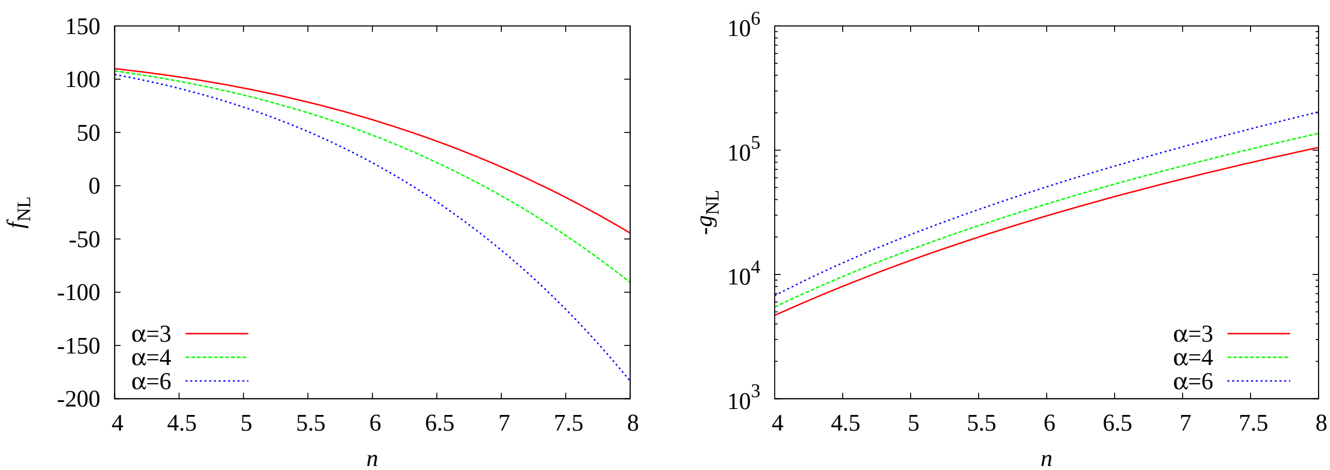

Figure 1: Plots of $f_{\rm NL}$ and $g_{\rm NL}$ as a function of n for several values of α. The values of s and $r_{\rm dec}$ are taken as $s = 0.05$ and $r_{\rm dec} = 0.01$.

[#4] Recently, a constraint on $g_{\rm NL}$ has been obtained for the case of a negligible $f_{\rm NL}$; the limit is $-3.5 \times 10^5 < g_{\rm NL} < 8.2 \times 10^5$ [33].

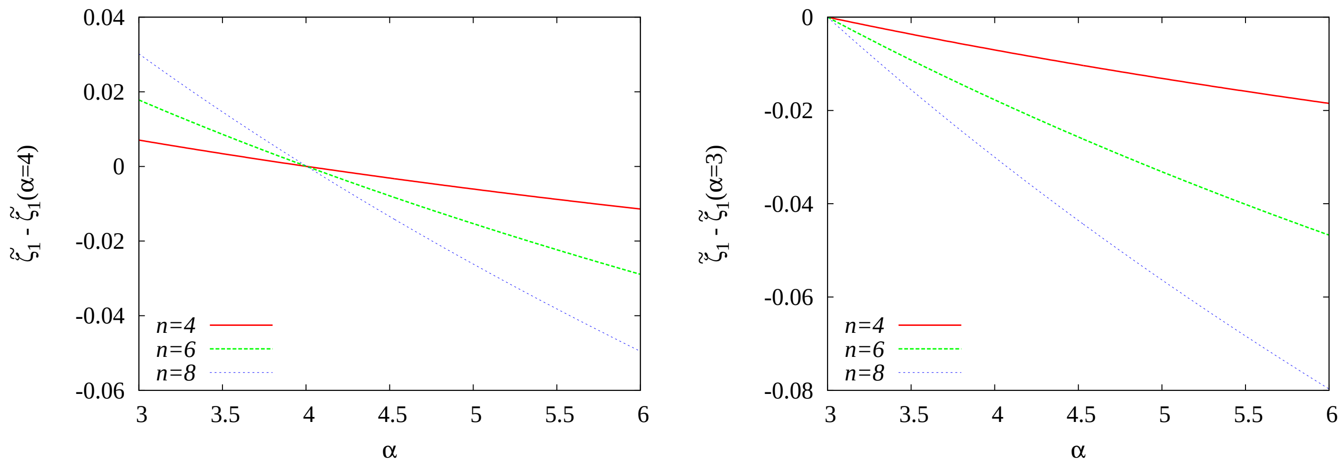

Figure 2: Plots of $\tilde{\zeta}_1 = \zeta_1/\zeta_1^{(\mathrm{quadratic})}$, which is normalized to ζ_1 for the pure quadratic case, relative to that for the cases with $\alpha = 4$ (left) and $\alpha = 3$ (right). The values of s and r_{dec} are taken as $s = 0.05$ and $r_{\mathrm{dec}} = 0.01$.

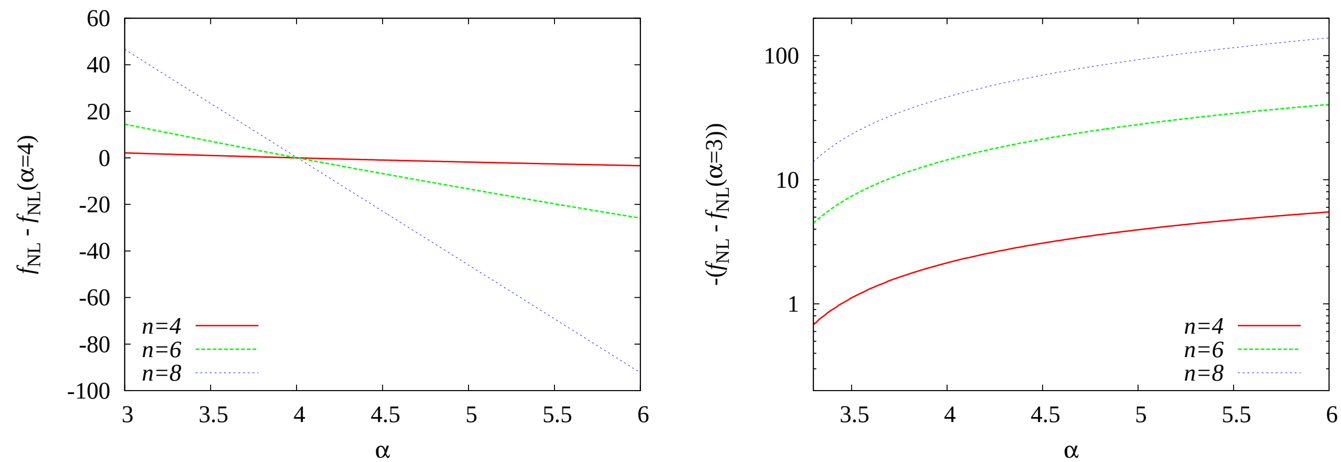

Figure 3: Plots of f_{NL} relative to that for the cases with $\alpha = 4$ (left) and $\alpha = 3$ (right). The values of s and r_{dec} are taken as $s = 0.05$ and $r_{\mathrm{dec}} = 0.01$.

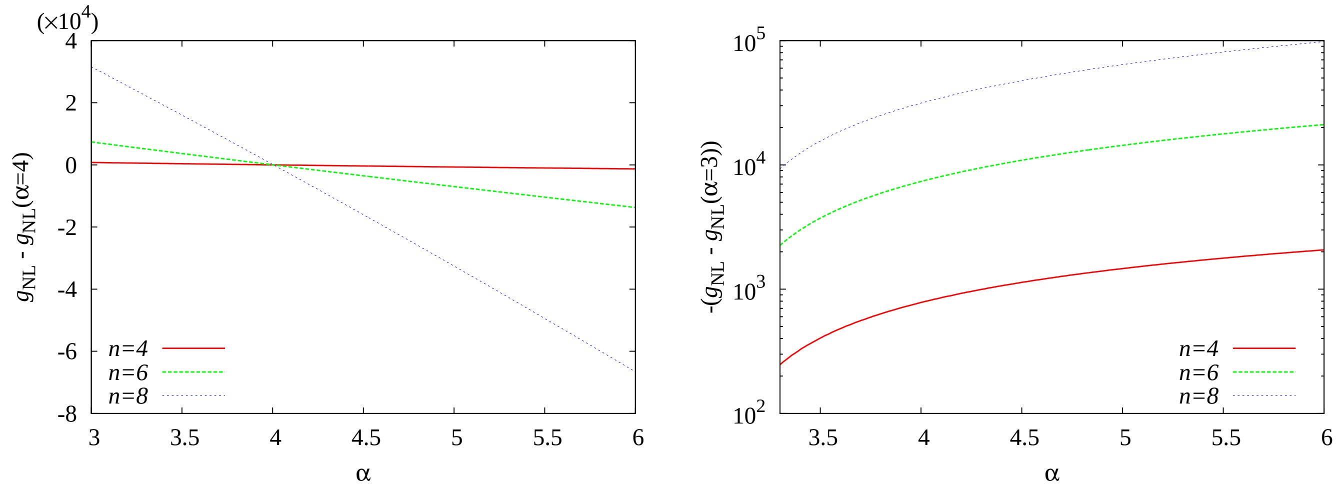

Figure 4: Plots of $g_{\rm NL}$ relative to that for the cases with $\alpha = 4$ (left) and $\alpha = 3$ (right). The values of s and $r_{\rm dec}$ are taken as $s = 0.05$ and $r_{\rm dec} = 0.01$.

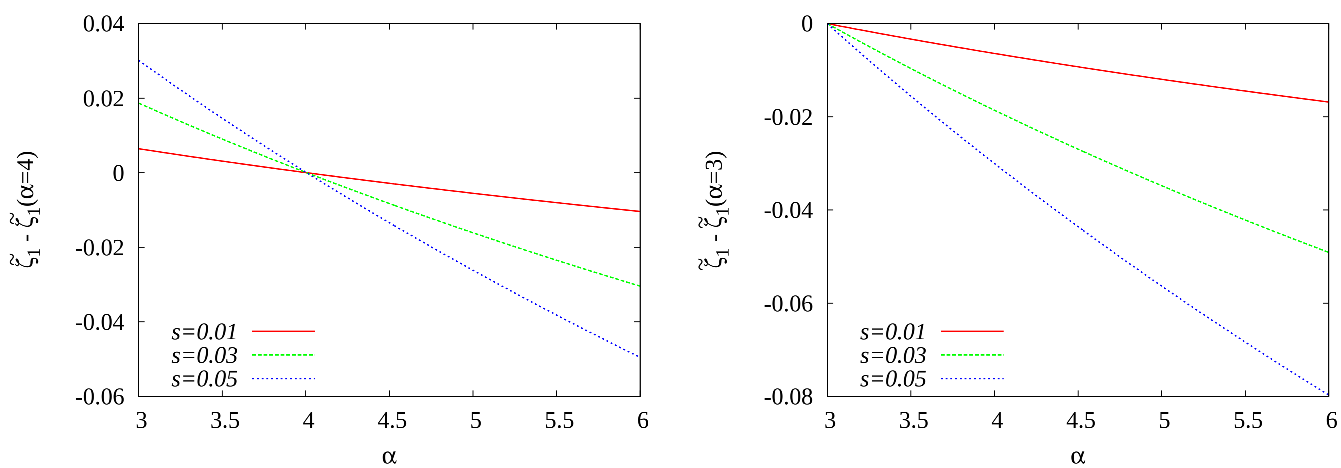

Figure 5: Plots of $\tilde{\zeta}_1 = \zeta_1/\zeta_1^{\rm (quadratic)}$, which is normalized to ζ_1 for the pure quadratic case, relative to that for the cases with $\alpha = 4$ (left) and $\alpha = 3$ (right). The values of n and $r_{\rm dec}$ are taken as $n = 8$ and $r_{\rm dec} = 0.01$.

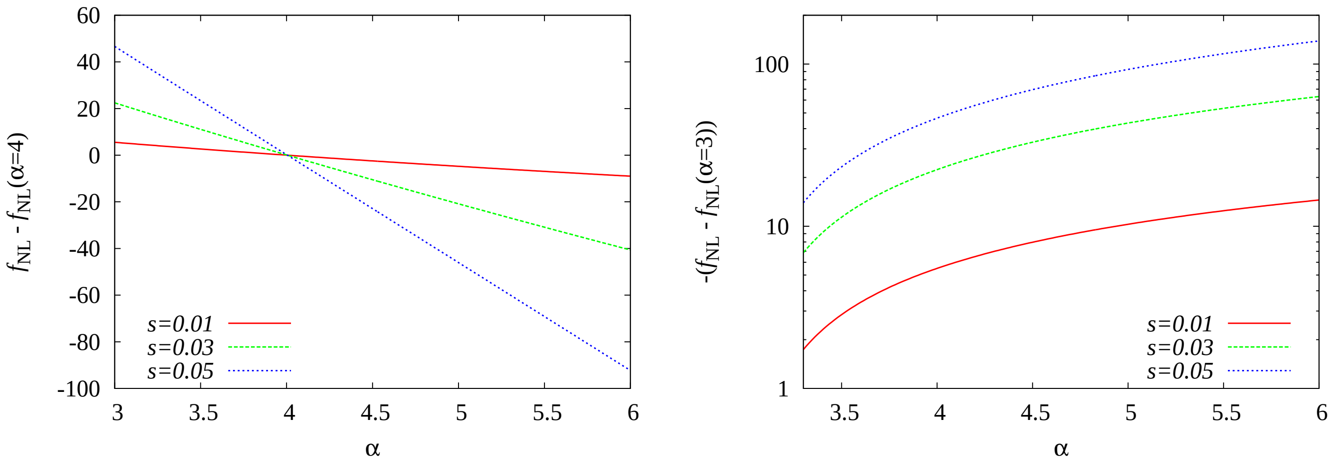

Figure 6: Plots of $f_{\rm NL}$ relative to that for the cases with $\alpha = 4$ (left) and $\alpha = 3$ (right). The values of n and $r_{\rm dec}$ are taken as $n = 8$ and $r_{\rm dec} = 0.01$.

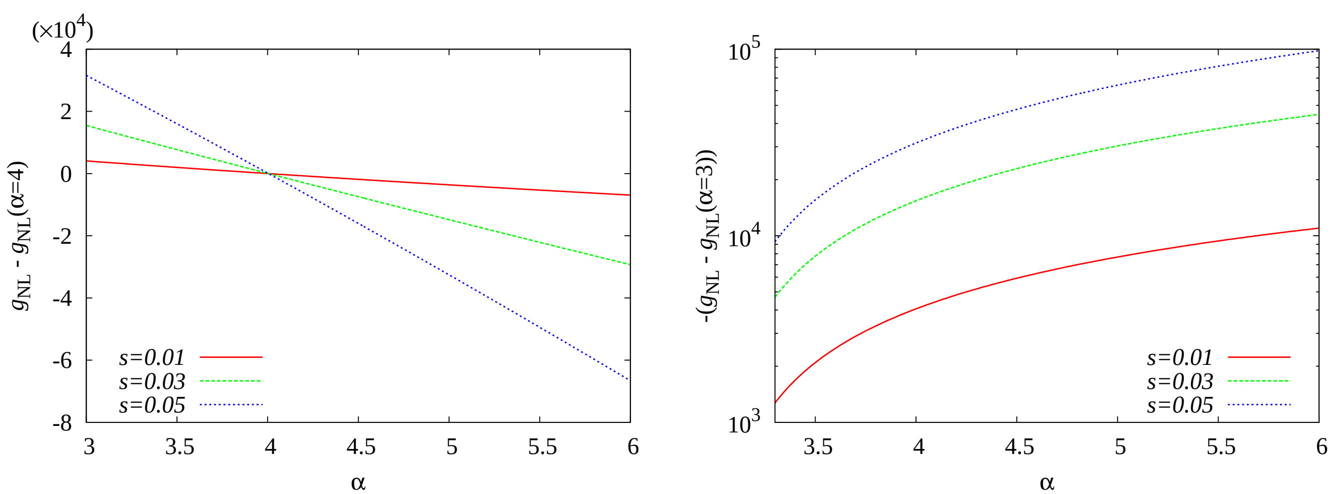

Figure 7: Plots of $g_{\rm NL}$ relative to that for the cases with $\alpha = 4$ (left) and $\alpha = 3$ (right). The values of n and $r_{\rm dec}$ are taken as $n = 8$ and $r_{\rm dec} = 0.01$.

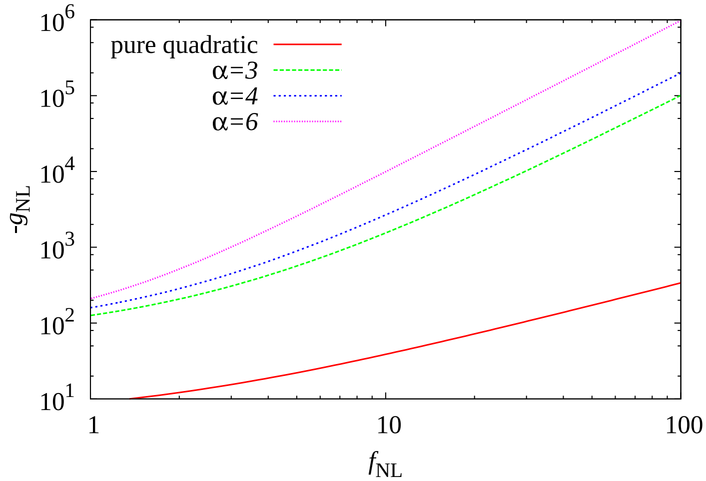

Figure 8: A relation between $f_{\rm NL}$ and $g_{\rm NL}$ for the case with $\alpha = 3, 4$ and 6. The values of s and n are taken as $s = 0.02$ and $n = 8$. For reference, a pure quadratic case with $\alpha = 4$ is also shown.

4 Conclusion

In this paper, we have investigated the effect of the background evolution on the curvaton non-Gaussianity, assuming a curvaton potential which slightly deviates from the quadratic one. Such a study is motivated by the possibility that after inflation, the inflaton keeps oscillating about its global minimum for a long time so that the curvaton could actually decay while the universe is still effectively matter dominated. More exotic temporary possibilities, such as a kination driven universe, could also be envisaged. Therefore we have considered an ideal background fluid with some generic equation of state, leading to a background evolution $\propto a^{-\alpha}$, with α a free parameter.

It turns out that the changing of the background to radiation, or equivalently, a change in the value of α, that takes place during curvaton oscillations, has by itself little effect on the perturbation. What matters is the nature of the background evolution before radiation domination finally kicks in, and as we show, it can lead to significant and potentially observable consequences. The non-linearity parameters $f_{\rm NL}$ and $g_{\rm NL}$, as well as the linear curvature perturbation ζ_1, depend on the nature of the background fluid. We find that the dependence on the background fluid becomes more pronounced as the deviation of the curvaton potential from the quadratic one increases. Typically, when replacing one background fluid with another, one induces effects on $f_{\rm NL}$ that are easily of the order of $\mathcal{O}(10)$ but could also be much larger, depending on the relative strength of the non-quadratic part of the potential.

An interesting issue is the relation between $f_{\rm NL}$ and $g_{\rm NL}$. We have showed that the relation depends both on the form of the curvaton potential and the background evolution,

which we find a rather surprising result. Hence measuring both $f_{\rm NL}$ and $g_{\rm NL}$, or both the bispectrum and the trispectrum, would yield information not only on the curvaton self-interactions but also on the equation of state of the background fluid. Since that is linked to dynamics in the inflaton sector, here arises a possibility of probing inflaton physics at the very end of inflation.

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